

# Comments on “On Bohm trajectories in two-particle interference devices” by L. Marchildon

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Marchildon’s arguments against my earlier work are refuted.

## I. INTRODUCTION

Marchildon [1] has attempted to show that claims made by me [2] and Golshani and Akhavan [3] that de Broglie-Bohm (dBB) mechanics and standard quantum theory (SQT) are incompatible in the case of certain two-particle interference experiments is wrong. I will restrict my comments strictly to his criticism of my work, and show that my basic conclusion stands.

## II. NON-ERGODIC PROPERTIES OF BOHMIAN MOTION

Let me begin by first pointing out that my conclusions are based on a very well known result in the study of ergodic problems in mechanics, namely that a decomposable dynamical system is non-ergodic, and that for such systems the space mean and time mean are not the same in general. SQT is necessarily always ergodic whereas Bohmian systems can be non-ergodic, as I will show. This is the basic reason for their incompatibility.

For the sake of clarity, let me define what is meant by a decomposable system. Let  $(M, \mu, \phi_t)$  be a dynamical system, where  $M$  is a smooth manifold,  $\mu$  a measure on  $M$  defined by a continuous positive density, and  $\phi_t : M \rightarrow M$  a one-parameter group of measure preserving diffeomorphisms. If  $f$  is a complex-valued function on  $M$ , its time mean is defined by

$$f^* = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=0}^{N-1} f(\phi_t^n x), \quad x \in M, n \in \mathbb{Z}^+ \quad (1)$$

and its space mean by

$$\bar{f} = \int_M f(x) d\mu \quad (2)$$

A dynamical system is ergodic if for every complex-valued  $\mu$ -summable function  $f$  the time mean is equal to the space mean:

$$f^* = \bar{f} \quad (3)$$

Now, let  $M$  be the disjoint union of two sets  $M_1$  and  $M_2$  of positive measure, each of which is invariant under  $\phi_t$ :

$$\phi_t M_1 = M_1, \quad \phi_t M_2 = M_2 \quad (4)$$

Such a system is called a decomposable system. A decomposable system is not ergodic. Conversely, a non-ergodic system is decomposable. Finally, a system is ergodic if, and only if, it is indecomposable. The interested reader is urged to look at the well known book on ergodic properties of classical mechanics by Arnold and Avez [4].

Having cited these well established fundamental results, I will first prove that dBB is non-ergodic whenever the two-particle wavefunction  $\Psi(x_1, y_1, x_2, y_2, t) = R(x_1, y_1, x_2, y_2, t) \exp \frac{i}{\hbar} S(x_1, y_1, x_2, y_2, t)$  has the following symmetries:

- (a) exchange or bosonic symmetry,
- (b) reflection symmetry about an axis (say the  $x$  axis so that  $x = \text{constant}$  is the axis of reflection symmetry),
- (c) translation symmetry of its phase  $S$  along that axis,

and in addition,

- (d) the initial positions of the pair of particles satisfy the constraint

$$x_1(0) + x_2(0) = \delta \quad (5)$$

where  $\delta \approx 0$  is a very small constant, and

(e) there are no more than a single pair of particles in the apparatus at any time.

(A realistic experimental arrangement in which these conditions can be simulated to a high degree of accuracy will be discussed in the next section.)

Proof: For any fixed value of  $(y_1, y_2)$   $S$  is then a function of  $(x_1 - x_2)$  only, and therefore it follows from the definition  $v_i(x_1, x_2) = \dot{x}_i = \partial_i S(x_1 - x_2)$  ( $i = 1, 2$ ) of Bohmian velocities that

$$v_{(1)}(x_1, x_2) + v_{(2)}(x_1, x_2) = \dot{x}_1 + \dot{x}_2 = 0 \quad (6)$$

and hence

$$x_1(t) + x_2(t) = x_1(0) + x_2(0) = \text{constant} \quad (7)$$

$(x_1(t) + x_2(t))$  is therefore a constant of motion.

Since the initial positions of the individual pairs in an ensemble can fluctuate randomly, let us assume that for every trial lasting a very small time interval around  $t_n$ ,

$$x_1(t_n) + x_2(t_n) = \delta_n \quad (8)$$

where  $-d/2 \leq \delta_n \leq d/2$ ,  $d$  being a negligibly small positive number compared to the (finite) range of  $x$  values accessible to the particles (to be identified with the slit width in the next section). Then, the symmetry axis  $x = \delta_n/2$  will fluctuate from one trial to another but will always remain bounded within the range  $-d/2 \leq x \leq d/2$  around  $x = 0$ . Whenever one particle is on the symmetry axis, its partner must also be on it according to (8), and therefore the  $x$ -components of their velocities must vanish on the symmetry axis according to (6). This means the trajectories will not cross the symmetry axis in any trial, and the Bohmian system is decomposable and non-ergodic. Q.E.D.

However, since the concept of trajectories does not exist in SQT, the corresponding system in SQT is indecomposable and ergodic. Nevertheless, SQT and dBB are by construction compatible in every conceivable case at the level of space means of observables. This is achieved in the following way. In SQT,  $|\Psi(x_1, x_2, \dots, x_m, t)|^2 \equiv R^2(x_1, x_2, \dots, x_m, t)$  gives the probability density of finding the  $m$  particles distributed in a particular fashion at a given instant of time if the particles in the ensemble were to be observed at that instant. In dBB one introduces a real statistical ensemble of particles with a probability density  $P(x_1(t), x_2(t), \dots, x_m(t))$  that is the same as in SQT. To be precise, dBB postulates that

$$P(x_1(t), x_2(t), \dots, x_m(t)) = R^2(x_1, x_2, \dots, x_m, t) \quad (9)$$

for all times  $t$ . This is guaranteed if it holds for the initial time by virtue of the continuity equation for  $P$  that is also assumed. Then, it can be shown [5] that for every observable  $\hat{O}$  in SQT (except the momentum  $\hat{p}$  which requires special treatment [7]) and the corresponding dynamical variable  $O$  in dBB,

$$\begin{aligned} \langle \hat{O} \rangle &= \int \dots \int \Pi_{i=1}^m dx_i \Psi^* \hat{O} \Psi \\ &= \int \dots \int \Pi_{i=1}^m dx_i O P(x_1(t), x_2(t), \dots, x_m(t)) = \bar{O} \end{aligned} \quad (10)$$

This demonstrates the equivalence between dBB and SQT for space means, i.e., averages over the (Gibbs) ensemble of possible states of the system at a given instant of time. In particular, the joint detection probability of two-particles defined as a space mean or ensemble average is given by

$$\bar{P}_{12} = \int_{D_1} \int_{D_2} dx_1 dx_2 R^2(x_1, x_2, t) = \int_{D_1} \int_{D_2} dx_1 dx_2 P(x_1(t), x_2(t)) \quad (11)$$

where  $D_1$  and  $D_2$  are the supports of the two detector faces.

The situation is different for a time ensemble when the Bohmian system is decomposable and non-ergodic. In the two-particle case under consideration, let  $x = (x_1, x_2)$ . Then

$$\phi_t^n x = \frac{1}{\delta(0)} (x_1(t_n), x_2(t_n)) \delta(x_1(t_n) + x_2(t_n) - \delta_n) \quad (12)$$

and the joint detection probability as a time mean is given by

$$P_{12}^{*'} = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=0}^{N-1} P(\phi_t^n x) |_{D_1, D_2} \quad (13)$$

which vanishes if the detectors are placed sufficiently asymmetrically about the  $x = 0$  axis (both on the same side of the symmetry axis, for example). This is in conflict with the SQT prediction (11) for  $\bar{P}_{12} = P_{12}^*$  which does not vanish for asymmetrically placed detectors. This concludes the proof of incompatibility.

We will now proceed to see if the conditions (a) through (e) can be met in an actual experimental set up, and to what degree of accuracy.

### III. EXPERIMENTAL DESIGN CRITERIA

The above conditions can be very well met by a careful design of the experimental arrangement. I proposed that a double-slit arrangement with momentum-correlated particles at the source, so that single pairs of these particles could pass through two narrow slits one at a time (condition (e)), would be appropriate. Therefore, Marchildon's comment at the end of page 3 following equations (7) and (8) that the "overwhelming majority of pairs are not simultaneously on the plane of symmetry" does not apply to this case.

There are, however, two other points of experimental design that need to be taken care of.

First, the wavefunction very close to the two slits of small width  $d = 2\delta$  is not plane, and therefore its phase is not a function of  $(x_1 - x_2)$  alone. This means that even if the particles at the slits satisfy the condition (8), when they enter the Fraunhofer region with translation invariance, their positions will no longer satisfy this condition. However, Marchildon [6] has shown that with spherical waves emanating from each point of the slits,

$$\frac{dx_1}{dt} + \frac{dx_2}{dt} = \frac{\hbar k}{mL}(x_1 + x_2) = \frac{v}{L}(x_1 + x_2) \quad (14)$$

where  $v = \hbar k/m$  is the velocity of the particles and  $L$  is the distance between the slits and the final screen or detectors, is a good approximation. Then it follows that

$$x_1(t) + x_2(t) = (x_1(0) + x_2(0))e^{vt/L} \leq de^{vt/L} \quad (15)$$

This shows that the exponential factor will grow from unity to the value  $e$  by the time the particles arrive at the detectors. As long as the Fraunhofer region sets in much earlier than this, the particles will enter this region satisfying the condition (8) to an excellent degree of approximation. The apparatus must be designed to take care of this. This can be done by a suitable choice of the slit width  $d$ , the separation  $a$  between the slits and the wavelength of the particles.

Second, it is also necessary to ensure that the initial wave packets do not spread appreciably within the apparatus of length  $L$ . Since

$$\frac{\sigma_t}{\sigma_0} = [1 + (\frac{\hbar t}{2m\sigma_0^2})^2]^{1/2} \quad (16)$$

for Gaussian wave packets (equation (17) in [1]), this means, for example, that the spreading will be small for electrons traversing a length  $L = 10^2$  cm with a velocity of about  $10^{10}$  cm/sec and initial width  $\sigma_0 \approx 2 \times 10^{-4}$  cm. These are neither theoretically impossible nor unrealistic values.

I must conclude by saying that the actual experiment that is being planned is with down-converted photons. This is necessitated by the experimental difficulty of producing a pair of momentum correlated electrons or neutrons. The experimental design is being done carefully to take all the above precautions into account. In fact, one advantage of using photon wavepackets is that they do not spread. The Bohmian trajectories for this case have been calculated and can be seen in Ghose *et al* [8].

### IV. ACKNOWLEDGEMENT

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